

## SOLUTION - PROBLEM OF THE MONTH, FEBRUARY 2018

*Congratulations to Jason Belanger, Mayank Jain and Piotr Laskawiec for submitting correct solutions to the February problem!*

**Problem.** Now that 2017 just ended and 2018 just began, here is a nice problem for you to think about.

Find a positive integer  $n$  such that the first four digits of  $n^3$  are 2018 and the last four digits of  $n^3$  are 2017. In other words,  $n^3$  has to have the following form

$$n^3 = 2018 \dots 2017$$

For additional bragging rights, prove that there exist infinitely many such positive integers.

Submit your solutions to professor Dan Ismailescu, Mathematics Department via email at dan.p.ismailescu@hofstra.edu, or bring it in person at 103C Roosevelt Hall.

### Solution

For  $x^3$  to end in 2017 one needs that  $x \equiv 9073 \pmod{10000}$ . On the other hand, it is not hard to find numbers whose cubes start with 2018. Two such examples are  $5866^3 = 201848801896$  or  $27228^3 = 20185858556352$ . Numbers that have both properties can then be constructed by combining two such numbers. Here are such examples:

586609073

5866009073

58660009073

586600...009073

272289073

2722809073

27228009073

2722800...009073